Mine Warfare Models

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1. Introduction.

A mine is basically a weapon that can't move and can only attack a target by blowing itself up, a rather primitive approach to warfare. Being required neither to move nor to project power at a distance, mines are relatively cheap; a mine may cost thousands of dollars while a missile or torpedo of equivalent destructive power would cost hundreds of thousands. Being cheap and available on the international arms market, mines can be employed in significant quantity by any country with even a modest military budget. They can be very effective. In 1950 during the Korean War, the minefield in Wonsan harbor inspired RADM Alan Smith to say,

> *The US Navy has lost control of the sea to a nation without a Navy, using pre-World War I weapons laid by vessels that were utilized at the time of the birth of Christ. (Milia, 1991).*

That minefield delayed the planned landing at Wonsan by over a week while 250 ships steamed back and forth outside the harbor. The United States Navy lost four minesweepers in the process of clearing it, and several other ships were also sunk or damaged (Hartmann (1979)).

About Iraq's use of mines in the Gulf War, ADM Arthur (COMUSNAVCENT) said

> *Iraq successfully delayed and might have prevented an amphibious assault on Kuwait's assailable flank, protected a large part of its force from the effects of naval gunfire, and severely hampered surface operations in the northern Arabian Gulf, all through the use of naval mines.(Mardola and Schneller, 1998)*

Even when the location and nature of the Iraqi minefields was revealed after the war, it took several months for the allied nations to clear them.

The first effective use of mines was by the Confederacy in the US Civil War; the "torpedoes" that Adm. Farragut damned at Mobile Bay were actually mines. Mines have been employed effectively in every major war since then. Hartmann (1979) gives a concise naval history as well as considerable technological information, or see Milia (1991). Mines will surely continue to be an important part of warfare. The availability of cheap microprocessors with low power requirements has given modern mines a technological advantage, and even mines designed decades ago have shown themselves to be effective in recent combat (Wettern (1991)).

Minefield models at several levels of complexity are needed to study, rehearse, and conduct this important component of warfare. These notes will review naval planning models, beginning with the simple and proceeding to the complex. For the most part these models make heavy use of the Theory of Probability, a natural consequence of the fact that neither side knows exactly what the other is doing in mine warfare. Reference will be made to models that are or have been in use by the US Navy, as well as to models that are hypothetical.

2. A Little Technology.

The earliest sea mines were contact mines. Contact mines are still in use, but they have three important disadvantages. Except in shallow water, one disadvantage is that they must be anchored to the seabed by a cable that extends nearly to the surface, making them vulnerable to

mechanical minesweeping. A second disadvantage is that the radius of action is limited by the target's presented width, and the third is that sea mines are most lethal when they detonate significantly below the target, rather than in contact with it. There are thus three powerful reasons for employing mines that can sense targets at a distance, so it should not be surprising that most modern mines are "influence" mines of this type. In water that is not too deep (roughly 200 feet, depending on charge weight and target), influence mines can rest on or near the seabed and still be a threat to targets on the surface. In deeper water they must either be moored or have some way of moving toward the target. The former choice makes the mooring cables vulnerable and the latter makes the mines expensive, so, given a choice, a minefield planner would prefer water that is not too deep. Figure 1 shows the options available as a function of water depth, including the possibility of a rising mine in deep water.

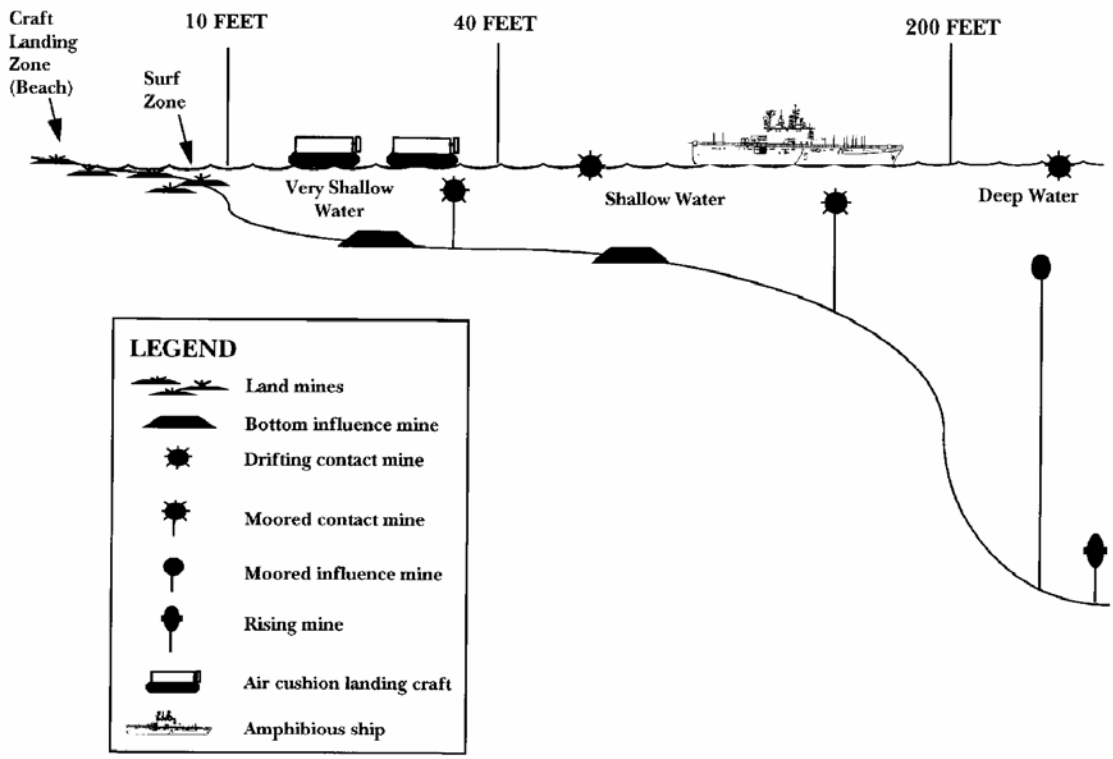

Figure 1: Mining options depend on water depth.

The three most common sensory phenomena are magnetism (the passage of a steel ship changes the local magnetic field), sound (ships make underwater noise), and pressure (there is a temporary decrease in pressure under the keel of a moving ship, proportional to the square of the ship's speed). The first two sensor types permit longer detection ranges than the third, but are subject to sweeping by minesweepers or helicopters that artificially create the magnetic/acoustic signatures characteristic of target ships. The advantage of the pressure sensor is that there seems to be no way to create the pressure effect except by having a large "guinea pig" ship pass over the mine, an awkward sweeping technique. However, the pressure sensor is subject to false alarms due to waves, so it is usually used in combination with others. Using a combination of

sensors also tends to frustrate minesweeping, as does the employment of other counter-countermeasures such as time delays or "counters" that detonate the mine only after it has been actuated a certain number of times.

Mines can also be countered by "hunting", by which is meant locating a mine by some mechanism (eyeball, sonar, laser,…) independent of the mine's sensors. Any "mine-like-objects" detected are examined more closely and, if judged to be mines, either avoided or destroyed. Hunting has an advantage over sweeping in that counter-countermeasures that work against sweeping are without effect, but hunting suffers from false alarms, a relatively low sweep width (particularly against bottom mines that are partially buried), and susceptibility to decoys. The proper division of effort between sweeping and hunting is one of the reasons for developing mine warfare models.

A third countermeasure is to cover a given area with such intense lethal effects that all mines contained in it are necessarily destroyed. This "destruction" alternative has the advantages that it can't be outwitted and that false alarms are not an issue, and the disadvantages that it is expensive and (of course) destructive. It is generally implemented by line charges or intense air strikes, and used only when minefields are both dense and unavoidable.

Water is denser and less compressible than air, so sea mines tend to have a much larger radius of action than land mines, particularly against targets subject to damage by shock waves. Sea mines are also harder to sweep and hunt than land mines, so mine warfare is an essentially different topic in the Navy, where mines are a potential show stopper, than in the Army, where mines tend to be viewed as a nuisance, albeit one that has to be planned for. An exception to this is the availability of artillery in conjunction with minefields on land. Artillery does well against concentrated targets, and since one countermeasure to minefields is concentration, the two measures can be particularly effective if used together. Naval minefields are rarely supported by artillery, although the WWI minefield in the Dardanelles is an exception to this.

Once planted, a minefield does not distinguish between friend, foe, or neutral. The Hague Convention of 1907, which was adopted by many nations after mines laid in the Russo-Japanese War caused extensive damage to merchant shipping, contains some rules designed to prevent damage to neutrals. Floating mines are essentially prohibited, and the existence of minefields threatening to neutrals must be published. Floating mines still occur, however, and the use of influence mines was not even anticipated in 1907. It remains true that most ships damaged by mines are neutrals, rather than combatants. The situation is even more serious on land, where there is widespread use of mines costing as little as $5 that remain dangerous for years after they are laid.

3. Campaign/Theater Level Models.

Large scale models of warfare are generally not built to study the details of minefield construction and countermeasures, but still need to represent mine warfare in some simple manner. The problem in such models is to retain the essence of mine warfare without including too many details, databases, or megaflops.

In a wargame, a "minefield" might be as simple as a prohibited region with an associated story line. The designer might announce, "Blue ships are not allowed to transit XYZ strait because of the presence of minefields". The implied model of mine warfare is that countermeasures are impossible, reconnaissance is perfect, and Red's logistic problems in creating the minefield are negligible. All of those statements might be false, but even so the model might be satisfactory. XYZ strait might be a shallow, easily mined area that, in the judgement of the wargame designer, would simply be avoided by Blue in the event of a conflict.

The players might also be allowed to create their own minefields if the game included realistic rules and constraints. A possible set of rules might be:

1) Red can construct only 10 square miles of minefield during the game, and each minefield requires the presence of some Red unit when it is created.

2) Except for minesweepers, any Blue unit is sunk immediately upon entering a Red minefield, and the outlines of the minefield are then revealed to Blue. Red units are unaffected by Red minefields.

3) Entry of any Blue minesweeper into a Red minefield will immediately reveal its outlines to Blue, and furthermore the minefield will disappear 48 hours later.

There would be similar rules for minefields created by Blue. These rules are probably overly simple, since they permit a single unit to create or counter a minefield. A clever player might create lots of long, thin minefields covering very little area that would effectively prohibit movement by the other player, an effective but unrealistic tactic that is permitted by the rules. Nonetheless, the rules are easily understood, easily implemented, and adequate for some purposes. They permit mine warfare to be "played" in a manner that is impossible if minefields are simply announced by the designer. A rough replication of what happened at Wonsan might happen within them.

The above sketches might be called "permission" models, since the central idea is an area where every unit either has permission to enter or not. While such models are useful for some purposes, an important idea is missing -- the idea that a ship might enter a minefield and still not be damaged. The fact is that *most* ships that enter real minefields are not damaged, so there is a danger of overstating the effectiveness of minefields if the possibility is ignored. Including it in a quantitative way will require the introduction of Probability, a characteristic of all the models that follow.

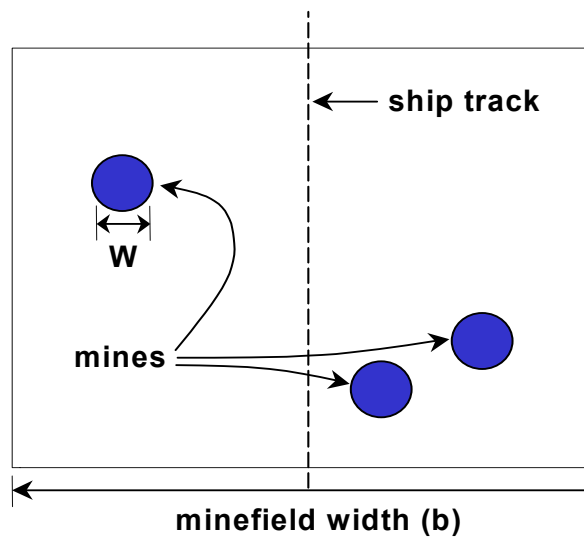

Figure 2: One ship, three mines

The US Navy's wargaming model ENWGS includes a mine warfare feature that is a permission model with one more level of detail: the number of mines M in the minefield. This number gradually decreases with time in ENWGS, either due to detonations caused by target ships or to minesweeping. ENWGS does not retain a location for each mine, but instead incorporates the assumption that a ship that travels a length L in the minefield during some time interval will actuate all mines in an area WL, where W/2 is the radius of action of each mine. If the minefield covers an area A, the probability that the ship actuates any randomly located mine is therefore WL/A. When running in its Monte Carlo mode, ENWGS computes for each ship the length L traveled in the minefield over the period in question, converts L to an actuation probability WL/A, and then generates a uniform random number U; if U is smaller than WL/A, the mine actuates and disappears. The ship also disappears unless it is a minesweeper. Alternatively, since all mines are assumed to be located independently at random within A, one could compare a single random number to the probability P_M that at least one mine is detonated by the ship:

$$P_M = 1 - (1 - WL/A)^{M.} \quad (1)$$

Comparing one random number to P_M is equivalent to comparing M independent random numbers to WL/A. Figure 2 shows as a dashed line the track of a ship that does not actuate either of three randomly located mines. Obviously the track and/or the mines could be rearranged so that one or both of the mines is actuated. Ignoring edge effects, the probability of that event is P_M.

Formula (1) requires the assumption that the mines are located independently at random in the minefield, an assumption echoed by most minefield models. There is an odd dissonance here: the platforms responsible for laying mines usually practice laying them accurately, whereas practically every minefield model begins by assuming that the mines are simply strewn about at random within the minefield. The reasons for this curious situation are worth a digression.

Seemingly a minefield planner would want to arrange his mines in such a manner as to leave no gaps in coverage, which would typically have them being evenly spaced on a single line perpendicular to the direction of ship traffic, rather than spaced randomly throughout the minefield. Laying mines in lines is also tactically convenient, so one would expect to encounter lines of mines in practice, rather than fields of them. In fact one does encounter mine lines in

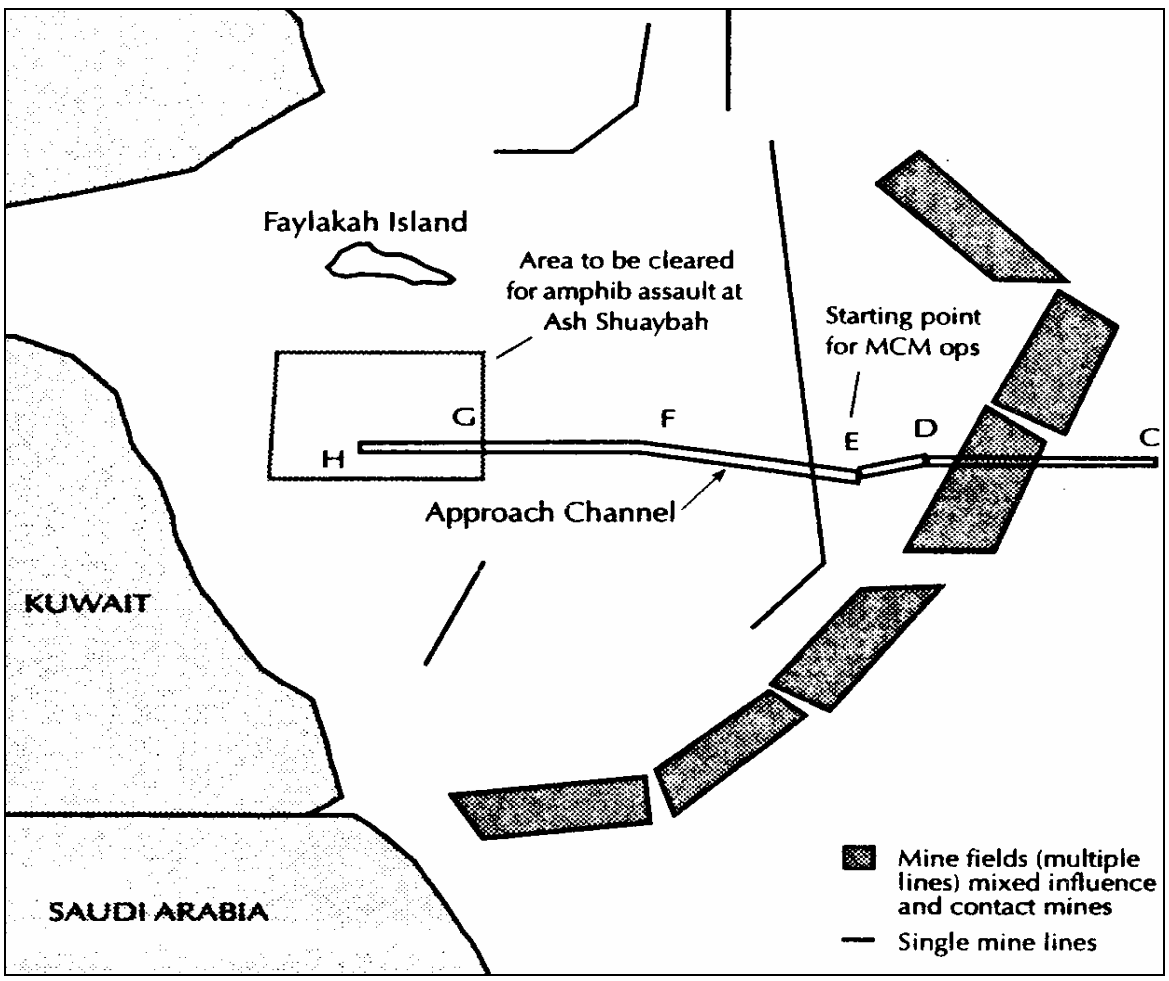

Figure 3: Desert Storm minefields

practice. Figure 3 shows the locations of the mine lines/fields laid by Iraq before Desert Storm. Even the areas shown as fields actually consisted of multiple lines. Incidentally (to digress a bit

within this digression), Figure 3 also makes it clear that the original US sweeping plan was in an area where there were no mines. The strikes on the Tripoli (moored mine) and Princeton (bottom mine) were the first indications that the minefields were actually located as shown, and the exact locations were not known until after the war. The Tripoli and Princeton paid for the lack of surveillance of minelaying operations (see Lyons, et al. (1993), from which Figures 1 and 3 were taken).

Even though Desert Storm mines were laid in lines, they were not laid in a single line. There are two advantages to the miner for not using a single line. One is the avoidance of fratricide among the mines or minelayers. The other is to complicate the MCM job, since mines in a single line are easy to sweep or avoid once the orientation of the line is discovered. At the end of World War II, Japanese Navy Captain Tamura was interviewed about the effectiveness of the Massive B29 drops of mines in Japanese waters (Navy, 1946). He said that the drops were on the whole very effective, but that

> *The mine laying planes always laid their mines in a simple row which made it easy for our lookout activities to analyze the plan and determine where the mines were and adopt effective countermeasures. It is necessary to vary the plan of laying occasionally.*

And so, partly because a minefield planner is already thinking of counter-countermeasures, a given "approach channel" like the one in Figure 3 is likely to include parts of several mine lines. Straighten out the approach channel into a long, narrow rectangle, and speculate about the cross-channel coordinates of the enclosed mines. They are unlikely to be evenly spaced for two reasons. First, the effective mines on a given line will not be evenly spaced because some mines are duds, some are deliberately configured differently from their neighbors, and because of navigation or timing errors in minelaying. Second, the positions on the various lines can reasonably be assumed independent -- how could they be coordinated when the minefield planner doesn't know exactly where the channel will be or whether it will be slightly crooked, like the one in Figure 3? The net result of superimposing the cross-channel coordinates of the mines on different lines, each with a different spacing, will be much closer to the cross-channel coordinates of a random minefield than to a minefield with regular spacing. In other words the random minefield assumption is robust to the kinds of deviations from the ideal of regularity that actually occur in practice. It is not true that mines are deliberately placed at random, but the effect is much the same.

This long digression has had the purpose of justifying the assumption of independence in equation (1). The independence assumption is not always so easily justified, and has caused considerable mischief when employed in the wrong circumstances. The assumption usually leads to simple, transparent computations, so it is often tempting to make it "as an approximation" to avoid some analytical complexity or database deficiency. If the assumption is substantially wrong, results can be misleading.

When equation (1) is applied to the first ship through a minefield, the left-hand-side is known as "Simple Initial Threat (SIT)". The threat to the second ship will not be as high. For one thing, the first ship might remove one of the mines (every ship gets to be a minesweeper once). This effect is handled in the ENWGS Monte Carlo simulation by decrementing the number of remaining mines, but there is an implied assumption about reality in proceeding with that method. The assumption is that the remaining mines have locations that are independent of the locations of the original mines, as if the passage of the first ship caused all of the mines to activate a little motor and move to a new position. The assumption is incorrect, since the remaining mines are a subset of the original mines and mines don't move. The falsity of the assumption might not be important if the second ship chose a track far away from that of the first

ship, but in fact the second ship is likely to take great pains to follow the first ship's track as closely as possible, especially if the first ship makes it through the minefield. That being the case, this second independence assumption is disastrous to the verity of the model for anybody wishing to explore the benefits of channelization, the most basic mine countermeasure. If the first ship actuates no mine, then the second ship's chances should be improved by the knowledge, but the ENWGS model gives the same chance to both. A naive user might conclude from experience with the model that the most basic countermeasure is actually ineffective.

It does not follow from the above comments that the ENWGS model is useless, but only that it should not be used to explore the benefits of channelization. ENWGS comes closer to reality than a simple permission model, and it does so without being excessively complicated, an important feature in a wargame where more important things than minefields must be represented accurately. This kind of situation is typical in studying mine warfare -- models are neither good nor bad in any absolute sense, but only for specific purposes.

4. Uncountered Minefield Planning Model (UMPM).

A more exact title for this section would say "almost uncountered", since planning is conducted in the expectation of channelization. The channel shown in Figure 3 was intended to avoid most of the Iraqi mines by using only a very small part of the mined area; all mines outside the channel have no chance as long as traffic sticks to the channel. Since the minefield must be planned without knowing where the channel will be located, channelization is an effective countermeasure to the extent that many channels are possible. Obviously the minefield planner would prefer a narrow constriction where the number of potential channels is small. Iraq had no such choice in mining the waters off Kuwait, but historically mines have tended to be utilized in straits and ports where traffic is naturally constricted.

Channelization never works perfectly because ships make navigation errors. Assume that ship navigation errors are independent random variables with a common density $f(x)$, with each error representing the ship's actual track relative to the channel centerline. The wider the error density, the wider the channel and the more effective the minefield. Typically $f(x)$ is assumed to be normal, but it will be treated generally in this section.

In section 3, actuation was assumed to be a matter of whether the ship came within $W/2$ of the mine. We now replace that assumption with one that realistically permits mine actuation to be uncertain. Specifically, let $a(x)$ be the probability that a mine actuates if the ship's closest point of approach is x, an "actuation curve" that can be determined by experimentation. The two functions $a()$ and $f()$ can be combined into one by convolving them to obtain $A(x)$, the probability that a ship attempting to follow a straight line will actuate a mine x away from the line:

$$A(x) = \int_{-\infty}^{\infty} a(x - u)f(u)du \qquad (2)$$

The effect of the convolution is that $A(x)$ has rounder corners than $a(x)$, as can be seen in Figure 4. In that figure $a(x)$ corresponds to a mine that always actuates if the ship comes within 100, while $A(x)$ includes the possibility of normal navigation errors with mean 0 and standard deviation 50.

In section 3, a ship was assumed to be damaged if and only if it actuated a mine. In actuality a ship may sometimes actuate several mines before being damaged, particularly if the mines have high sensitivity settings. Preventing these "wasted fires" is one of the main goals of the minefield planner, so it is essential to have a model that at least recognizes the idea. Let $d(x)$ be the "damage curve", the probability that a detonating mine will damage a ship at distance x, and let $D(x)$ be the probability that a ship attempting to follow a line will actuate a mine located x away from the line and be damaged by it. Then

$$D(x) = \int_{-\infty}^{\infty} d(x-u)a(x-u)f(u)du \qquad (3)$$

In principle the first actuation may not lead to detonation because of the presence of counter-countermeasures that require multiple actuations. These possibilities are being ignored in this section because no countermeasures are expected, hence the simple multiplication of $a(x-u)$ by $d(x-u)$ in (3). The difference $A(x)$-$D(x)$ is the probability that the mine actuates and does no damage, in other words the probability of a wasted fire.

Now consider a group of n ships that attempt to transit a line located a distance x away from a mine. All transits are independent when x is given because of the assumption about navigation error, so the probability that the mine actuates is $1-(1-A(x))^n$. The probability of damage given actuation is $D(x)/A(x)$, so

$$R_n(x) \equiv \text{P(1 out of } n \text{ ships is damaged by a mine at } x)$$
$$= D(x)\{1-(1-A(x))^n\}/A(x) \qquad (4)$$

The only other possibility is that no ships are damaged, since a mine can only detonate once.

Assume next that the ships attempt to follow a line near the center of a rectangular minefield of width b, as in Figure 2, and consider the effect of the first mine encountered, the mine nearest the horizontal line of ingress. The unconditional probability R_n^* that one of the first n ships is damaged by the first mine is just the average value of $R_n(x)$ across the breadth of the minefield:

$$R_n^* = \frac{1}{b}\int_{-b/2}^{b/2} R_n(x)dx \qquad (5)$$

Formula (5) also applies to the second and subsequent mines, possibly with a reduced value for n. So far the UMPM inputs have been an actuation curve, a damage curve, a navigation error density, a minefield width, and a number of ships. The computational effort required is mostly the numerical integrals required to evaluate (2)-(5). With the ENWGS assumptions, R_n^* would be given by P_n (equation (1) with $M=n$). Thus the main difference between UMPM and ENWGS is in the way R_n^* is calculated.

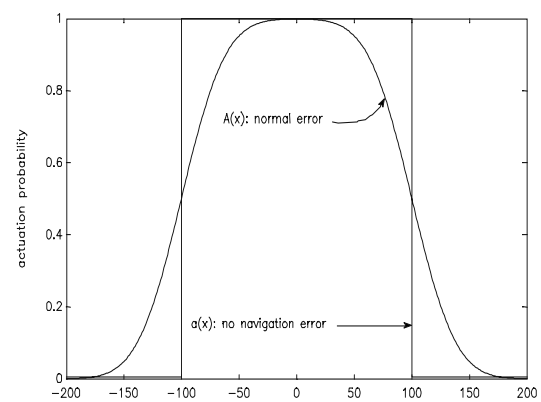

Figure 4: Actuation probability versus distance (x) from mine to intended track.

The numbers R_n^* turn out to be all that is necessary to analyze a minefield of multiple mines, even though only a single mine has been considered so far, but first it is necessary to consider quantitative goals for the minefield planner.

It is not obvious what Measure of Effectiveness (MOE) to use in planning a minefield. Simple Initial Threat is widely used, but *SIT* gives no clue to the threat to following ships, which can be much smaller than the threat to the first (imagine a minefield with one big mine). There has been some debate within the US mine warfare community over exactly what statistics are worth looking at when designing a minefield, the result being that UMPM computes and displays multiple MOE's for the inspection of the planner. For a hypothetical input number n of transiting ships, these include

1) "Threat profile": The probability that the ith ship is damaged by a mine, i=1,…,n. For i=1 this is *SIT*.

2) "Casualty distribution": The probability that k out of n ships are damaged, k=0,…,n. For i=0 the probability is called the "catastrophe probability"[1], catastrophe being from the viewpoint of the minefield planner.

3) "Stopped penetrator distribution".[1] For an additional user input "number of casualties after which no further transits will be attempted", UMPM outputs the probability that i ships will penetrate; that is, that i out of n ships will neither turn back nor be damaged by mines, i=0,…,n.

Each of these quantities has a similar method of computation in UMPM. The computational details are given below only for the casualty distribution (see Odle (1977) for the others). There is no loss of generality if one imagines that all n ships attempt to transit the minefield in a compact group that is gradually reduced in size as additional mines are encountered. Let $x(m,k)$ be the probability that k ships are still alive (undamaged) after the group has passed the first m mines, for $0 \le k \le n$. Then

$$x(m+1,k) = R_{k+1}^{*}x(m,k+1) + (1-R_k^{*})x(m,k);\ 0 \le k \le n, \qquad (6)$$

except that the first term is missing if $k = n$ because n is the total number of ships. Equation (6) expresses the idea that, if k ships remain after m+1 mines have been passed, then there must have been either k+1 or k ships alive after passing m mines. In either case, the probability that the $(m+1)^{st}$ mine does no damage is assumed to depend on the number of remaining ships, but not on the mine index. Since $x(0,k) = 0$ for $0 \le k < n$ and $x(0,n) = 1$, equation (6) can be used to calculate $x(1,k)$ for all k, then $x(2,k)$ for all k, etc., until finally $x(M,k)$ for all k is obtained. The casualty distribution is then $x(M,n-k)$; k=0,…,n.

The assumption in (6) that damage by (say) the second mine has the same probability as damage by the first is questionable, even when the number of ships is held constant. The reason is that the Theorem of Total Probability requires the damage probabilities in (6) to be <u>conditional</u> on the number of surviving ships, and there is information about navigation errors in the mere fact of survival. Ship navigation errors relative to the channel centerline are all independent *a priori* by assumption, but they are not independent under the condition of no damage by the first mine. In particular, the probability that the navigation errors all happen to be approximately equal ought to be relatively high under that condition, since one reasonable explanation of no damage to a group of ships is that they all happen to follow nearly the same lucky track. If navigation errors at the second mine are the same as at the first, use of (6) is therefore unjustified, strictly speaking. The simplest way out of this analytic crisis is to assume that ships wander about the centerline of the channel, so that navigation errors at the successive mines are independent even for the same ship. The resulting minefield analysis is known as "semi-configured", in contrast to the "fully-configured" case where ships travel in straight lines and the navigation error is the same at every mine, as illustrated in Figure 2. Thus UMPM is a semi-configured model. The fully-configured case is a much more difficult analytical problem in spite of its seeming simplicity.

Another way of putting it is that UMPM models the number of alive ships k as a Markov chain with a transition every time the group of surviving ships encounters a mine, with each transition being to either k or k-1 according to (6). The Markov assumption requires the ship's tracks to wobble about the centerline.

Some of the UMPM calculations are incorporated in sheet "UMPM" of workbook *MineWar.xls*.

[1] This term was coined by Jim Horrigan. Interest in stopped penetrator distributions is also due to him.

4.1 Simple extensions to UMPM

Several important phenomena can be included in semi-configured calculations by making minor modifications to the UMPM algebra.

If every mine has a reliability R that represents the probability that the mine is functional, independent of all other mines, then it can be accounted for by multiplying the damage curve $d(x)$ by R. The effect of this will be to multiply $D(x)$ and R_n^* by R. All measures of minefield effectiveness will be affected adversely.

A probability actuator is a counter-countermeasure that actuates the mine with probability ACT when the physical signals that would otherwise suffice for actuation are received, independently on each occasion until the mine finally actuates. To account for it, multiply the actuation curve $a(x)$ by ACT. This will have the effect of multiplying both $A(x)$ and $D(x)$ by ACT and thereby reducing R_n^* substantially for small n, or less so for large n. This will decrease SIT, but will increase the threat to later arriving ships.

Minefields are typically planned by inputting a fixed number of mines M and seeing the consequences, but the UMPM calculations can be easily adapted to the case where the number of mines is random. Let $P(m)$ be the probability that m mines will actually be present, m=0,…,M, and let $x(k)$ be the probability that k ships out of n will survive. Then

$$x(k) = \sum_{m=0}^{M} x(m,k)P(m) \qquad (7)$$

Since UMPM automatically computes $x(m,k)$ for m=1,…,M-1 in the process of computing $x(M,k)$, the data required for (7) are already available, and $x(n-k)$ is the desired probability of k casualties (but see exercise 14).

It may be desirable to include the possibility that a damaged ship will actuate additional mines, in contrast to the UMPM assumption that damaged ships sink immediately. If every damage incident results in sinking the victim with probability S, independently of all other incidents, then the casualty distribution can still be obtained by Markov chain calculations. The state space must be changed to include "ghost" ships (damaged but not sunk) as well as "virgin" ships, however, so these calculations would require a revision of the UMPM program, rather than a simple manipulation of inputs (see exercise 3).

4.2 Essential problems with UMPM.

It was mentioned in section 3 that UMPM is a semi-configured model: mines don't move, but ships are assumed to wobble enough in the channel to justify the independence assumption required in (6). A fully-configured analysis would probably come closer to reality, but the required modifications to UMPM would be complex. While this may be an example of a problem that is insoluble but not serious, there are also some serious problems. Chief among these is UMPM's use of a "pre-averaged" actuation curve.

Suppose that half of the ships are of type 1, with actuation curve $a_1(x)$, while the other half have actuation curve $a_2(x)$. Let $a(x) \equiv .5a_1(x)+.5a_2(x)$, so that $a(x)$ represents the probability that a randomly selected ship actuates a mine. Is there anything wrong with simply running UMPM with the single actuation curve $a(x)$? There would be nothing wrong with doing so if every ship somehow mutated between mines so that its type was independently selected for each mine, but unfortunately a ship's type remains fixed throughout the transit. Displacement, magnetic moment, speed, and noisiness are all important determinants of the actuation curve for which there is no reason to expect any fluctuation during a transit. The independence assumption that UMPM requires is not true in these circumstances, and the error involved in using it can be significant. For an extreme example suppose that $a_1(x) = 1$ and $a_2(x) = 0$ for all x, possibly

because the mines are magnetic and type 2 ships have no magnetic moment. Also suppose $d_1(x) = d_2(x) = 1$ for all x. The planner's object is to make $SIT = .9$. Using $d(x) \equiv 1$ and $a(x) \equiv .5$ in UMPM would lead to the conclusion that 4 mines are required. In actuality *SIT* cannot be made larger than .5 no matter how many mines are used, and only one mine is required to do so. This is an extreme example, but the effect can be significant even in practical situations.

Pre-averaging was described above as a problem caused by the variability of ships. The variability of mines causes similar difficulties. $a_1(x)$ might be for a mine with high sensitivity while $a_2(x)$ is for a mine with low sensitivity. The UMPM calculations assume implicitly that sensitivity is independently determined for each interaction with a ship, which isn't true. If mines differ significantly from each other, UMPM's predictions may be wrong. Here are some reasons why mines might differ from each other:

1) Magnetic mines that lie on the bottom, a common type, typically measure only one component of the magnetic field. Therefore the actuation curve depends on the orientation of the mine when it hits the bottom. The orientation is random, but does not change with time.

2) The mines might be of different types. Mixed minefields are not uncommon.

3) Tactical parameters such as sensitivity or actuation probability might be deliberately varied by the minefield planner.

4) Production variances. Two mines of the same type with identical settings will in reality perform differently.

To summarize, UMPM correctly handles the fact that the location of a mine does not change between transits, but all other mine characteristics are necessarily pre-averaged into the actuation curve. Mine location is surely the most important property to configure, and in this sense UMPM wisely devotes computational effort to that feature (in fact, configuration of mine location is the main difference between UMPM and the ENWGS model). Nonetheless, mines have many properties other than location that, while random, are not *independently* random for each mine-ship interaction. The resulting deviations from reality can be significant.

4.3 Monte Carlo simulation.

UMPM is an analytic model, whereas the ENWGS model of section 3 is a Monte Carlo simulation. Analytic and Monte Carlo methods are two essentially different approaches to probability problems that compete with and complement each other in many areas, minefield analysis being one of them. Analytic methods exploit independence assumptions to produce formulas like (6) that make computer implementation efficient. Monte Carlo methods appeal directly to the idea of probability as a long-run frequency, using a random number generator to determine random quantities in a repeated experiment. Monte Carlo methods do not require insights like (6), but they do typically require long computer run times to determine accurate results.

Appendix A is a flow diagram of a Monte Carlo simulation that parallels the UMPM assumptions except that it is fully-configured. It measures the casualty distribution $C()$ by making *REP* replications of an experiment where n ships transit a minefield with M mines. The number of casualties *CAS* is accumulated in the appropriate cell of $C()$ at the end of each replication. To parallel the UMPM calculations, the actuation question in Appendix A would be answered by finding the distance x between mine j and ship i and then testing a uniform random number against $a(x)$. Given actuation, the damage question would test another uniform random number against $d(x)$. The convoluted functions $A(x)$ and $D(x)$ are not required, but the density function of navigation errors is still needed because navigation error needs to be set for each ship in the "set ship properties" block. This navigation error is set only once for each ship in each

replication, which is why the simulation is fully-configured.

If *REP*=10,000, then $C(K)\pm.01$ is at least a 95% confidence interval on the true probability of *K* casualties, sufficiently accurate for most minefield planning. Modern (1995) computers are fast enough to do 10,000 iterations of Appendix A's logic in a few seconds, so an evaluative tactical decision aid could be based on Monte Carlo simulation (see Mullens (1993), Washburn (1995)).

Monte Carlo simulation could also potentially deal with pre-averaging problems. The actuation curve could depend on ship properties such as magnetic moment or on mine properties such as orientation, each property set once per replication. The damage curve might depend on ship displacement or some better measure of ship hardness. However, it is not just a matter of revising Appendix A, since these changes would require either a more extensive data base or a better physical understanding. UMPM requires only one actuation curve for a given ship-mine combination. To make that curve "depend on" the mine's orientation requires either an expansion in the amount of data that has to be measured and stored (8 orientations would require 8 times as much data, etc.), or else some method of adapting a single actuation curve to specific situations. An example of the latter approach would be to argue that magnetic actuation distance is proportional to the cube root of ship displacement[2], in which case one actuation curve will suffice for all ship displacements.

One could go further. UMPM and all of the minefield planning models discussed so far settle actuation questions at the closest point of approach using an actuation curve. One could dispense with the actuation curve, since it is in actuation that most of the pre-averaging problems arise. The idea would be to gradually move each ship along its track, use a physical model to predict influences, and imitate the mine's signal processing to decide when actuation occurs, if ever. In fact such detailed simulations already exist, the Total Mine Simulation System (TMSS) being one of them. The trouble is that simulations like TMSS are so slow that they cannot be used for tactical purposes. A good minefield planning model must be a compromise between the twin goals of accuracy and speed.

5.0 Mine Countermeasures (MCM).

In the rest of these notes the ships that are the target of the minefield will be referred to as "transitors". The MCM ships (or helicopters) that employ countermeasures will be called "minesweepers" or simply "sweepers", a term that is meant to include hunting as well as sweeping.

MCM planning has some features that make it more difficult than minefield planning. Minefields are usually laid in secrecy, so the location and even existence of the minefield may be initially unknown to MCM forces. If at least the identity of the miner is known, it may be possible to make some inferences about the <u>type</u> of mines to be expected, but even in that case the mines will still have unknown sensitivity, unknown mine count, and unknown delay arming adjustments. In the face of all this uncertainty, MCM forces must make clearance plans and eventually decide when to say "all clear".

The MCM problem has so much uncertainty that one might expect clearance plans to be sequential. That is, one might expect to encounter rules of the form "stop sweeping after five successive sweeps have detonated no more mines", or "hunt for 8 hours, determine the identities of any mines found, and then, depending on results, either continue to hunt or switch to sweeping". Sequential plans tend to be robust compared to nonsequential plans such as "sweep the field seven times" or "hunt 8 hours and then sweep 8 hours", so the use of sequential plans in

[2] The argument would be that the magnetic disturbance seen by a mine is proportional to ship displacement and (in the far field) inversely proportional to the cube of distance from the ship (Hartman, 1979, p 115).

MCM would seem natural. Sequential plans are employed in MCM, but not formally; that is, they are never known in sufficient detail that they can be written down in the manner of the two quoted fragments. Formal MCM plans turn out to be nonsequential plans developed with the aid of computer programs that begin by asking questions like "what are the dimensions of the minefield?", "what kind of mines are you trying to sweep?", and "what is the mine sensitivity?". Such nonsequential plans are easy to derive, easy to communicate, and easy to measure. On account of these virtues, nonsequential plans are dominant in practice in spite of the theoretical and intuitive virtues of sequential plans.

The primary MCM planning tools for the US Navy are a pair of computer programs that settle most of the uncertainties discussed above by requiring the operator to provide an input. NUCEVL (Non Uniform Coverage EVaLuator) is an evaluation tool that asks the operator to provide a sweeping plan and then outputs the fraction of mines at location y that will be swept at least k times, for operator selected values of y and k. The idea is that the operator inspects the output, decides whether the numbers are sufficiently large, and then possibly revises the plan. UCPLN (Uniform Coverage PLaNner) reverses the procedure by asking the operator to input the required sweep probability, and then outputs the uniform sweeping plan that just barely meets the requirement. Both programs are based on the work of R.K. Reber (1956).[3] The minefield is modeled more or less as in UMPM, except that minesweepers attempt to follow different paths and can't be sunk.

NUCEVL and UCPLN accept only "squared-off" actuation curves of height B and width A; that is, it is assumed that $a(x)=B$ if $|x|<A/2$, else $a(x)=0$. This approximation is mostly because the computer programs are automations of procedures that were at one time done by hand. Either way it is necessary to have a database that takes mine type and sweep type as input and produces an actuation curve as output. A database of curves is awkward for a manual procedure, particularly if the curve has to be used for further computations; it is much simpler to store and use two numbers A and B. The need for replacing curves by numbers disappears in the computer automation, but nonetheless the A/B approximation continues to be employed. Mine warfare is just one example of an area where concepts and procedures that were at one time forced on analysts by computational necessity persist long after the limits are relaxed.

Even though current tactical decision aids are nonsequential, it is worth exploring how sequential aids might be constructed. The decision to allow transitors into the swept minefield depends critically on the estimate of the number of mines that remain after sweeping. The quantification and sequential revision of such estimates are the subject of the next section.

5.1 Residual mines.

NUCEVL and UCPLN do not ask the user to guess the number of mines initially in the minefield, and provide no information about the residual number of mines after sweeping. The avoidance of reference to this seemingly vital number is actually natural, since the user is likely to be highly uncertain about it. The missing estimate causes no problem as long as the only quantity of interest is the fraction of mines cleared, but eventually a judgement that the minefield is "sufficiently safe" for transitors will have to be made. At that point an estimate of the number of mines remaining is required.

Suppose, for example, that a minefield is swept in such a manner that every mine is removed with probability .5, and that the number of mines removed Y is observed to be 4. How many mines are left? One could argue that there must be 4 left, since as many mines were not removed

[3] Reber was an early and prolific writer on probabilistic models of mine warfare. His work includes some sequential mine clearance ideas that he referred to as "observational rules".

as were removed. But this is a crude guess, since the number of mines remaining is clearly random, and besides it may seem odd that the number of mines remaining should increase with the number removed. One could argue just as effectively that removing mines should cause the number remaining to decrease, rather than increase. This paradox can be resolved by assuming a prior distribution for the number of mines and applying Bayes Theorem. A prior distribution is required if statements about the residual threat of the minefield are to be made; information about the fraction of mines removed is insufficient, in itself, for making an assessment of the number of mines remaining.

The rest of this subsection deals with a particular "Katz" class of probability distributions. If Excel workbook *MineWar.xls* is available, the reader may wish to open it up to sheet "Katz" before continuing. Formulas (10)-(12) below are built into that workbook, along with some graphics. In addition to the specific example given below, the reader can experiment with other inputs.

Let M be the initial number of mines, and let x_j be $P(M=j)$; $j\geq 0$. The prior distribution consists of x_0, x_1, etc. In principle any prior distribution can be used, but it turns out that there is a particular 2-parameter class of distributions with convenient analytic properties in this situation. This is the Katz class where the ratio formula $x_{j+1}/x_j=(\alpha+\beta j)/(j+1)$ holds for $j\geq 0$. The two parameters α and β must be such that $\alpha>0$ and $\beta<1$ (otherwise the sequence diverges), and in addition the ratio $-\alpha/\beta$ must be a positive integer if $\beta<0$. When $\beta<0$ the distribution is a binomial distribution with $-\alpha/\beta$ trials and $-\beta/(1-\beta)$ success probability. α/β need not be an integer when $\beta>0$, but if so then the distribution is the negative binomial distribution characteristic of counting the failures until the α/β^{th} success in repeated trials where the failure probability is β. When $\beta=0$ the distribution is Poisson with mean α. Katz (see Johnson and Kotz (1969)) showed that the probability generating function is $E(z^M)$ [4]$=((1-\beta z)/(1-\beta))^{-\alpha/\beta}$, from which all moments can be derived. For example $E(M)=\alpha/(1-\beta)$ and $\mathrm{Var}(M)=\alpha/(1-\beta)^2$. The probabilities themselves are easily generated by taking advantage of the fact that $x_0=E(0^M)=(1-\beta)^{\alpha/\beta}$; the ratio formula then determines x_1, x_2, etc. This Katz class of distributions has sufficient flexibility to reasonably approximate most unimodal priors for M.

Let $M'=M-Y$ be the number of mines remaining after sweeping, and suppose that sweeping is to the level p; i.e., every mine is independently removed with probability p. Let $q=1-p$. Then

$$P(M'=j\,|\,Y=y)P(Y=y)=P(Y=y\,|\,M=y+j)P(M=y+j)=\binom{y+j}{y}p^y q^j x_{y+j};\ j\geq 0, \quad (8)$$

since $P(Y=y|M=y+j)$ is the binomial probability of y successes out of $y+j$ trials. Let $x_j^*\equiv P(M'=j|Y=y)$ be the posterior distribution of the number of mines remaining. Then, taking the ratio of successive terms in (8),

$$x_{j+1}^*/x_j^*=\left[\frac{(y+j+1)}{(j+1)}\right]q\left[\frac{\alpha+\beta(y+j)}{(y+j+1)}\right] \quad (9)$$

The first [] factor is a ratio of combinatorial coefficients, and the second is by assumption x_{y+j+1}/x_{y+j}. The two $(y+j+1)$ factors cancel, so (9) is again a linear function of j divided by $(j+1)$. Thus the posterior distribution x_j^* is of the same Katz type as the prior distribution x_j, except that

[4] Here and throughout, E() denotes the expectation operator.

the revised parameters are

$$\alpha'=q(\alpha+\beta y),\ \beta'=q\beta. \tag{10}$$

Note that α'/β' will be an integer if α/β is, since $\alpha'/\beta'=\alpha/\beta+y$. The expected number of mines remaining increases with y if $\beta>0$ or decreases with y if $\beta<0$; depending on the prior distribution, either of the above arguments about dependence on y may be dominant. The Poisson case where $\beta=0$ is on the boundary; the posterior distribution still depends on p, of course, but not on y.

Since the posterior distribution is in the same class as the prior, it can itself be revised by further sweeping in the same way, so one might have α'', β'' for the distribution following a second search, etc. The appeal of the class is that the effect of sweeping is a simple revision of two parameters.

If each remaining mine will damage the first transitor with probability t, then the probability that the first transitor is not damaged is $E((1-t)^M)$, where M has whatever Katz distribution is characteristic of the amount of sweeping that has been done. Substitute $1-t$ for z in the probability generating function to get

$$SIT = g(\alpha,\beta,t) = 1-\left(\frac{1-\beta(1-t)}{1-\beta}\right)^{-\alpha/\beta} \tag{11}$$

By substituting (α,β) values appropriate to the amount of sweeping that has been done, SIT at any point can be determined. If $\beta=0$, replace (11) by the limiting case for small β: $g(\alpha,0,t)=1-\exp(-\alpha t)$.

One could also use (11) to forecast the SIT associated with a given clearance level p of an (α,β) minefield <u>without</u> knowing the number of mines removed. This forecast would be useful in planning the amount of sweeping to be done. The clearance level essentially reduces the threat of each mine from t to qt, so the forecast SIT before the number of swept mines is observed is $SIT'=g(\alpha,\beta,qt)$. In connection with planning it may be desirable to solve this for the q corresponding to a given SIT'. The solution is

$$1-p=q=\frac{1-\dfrac{1-(1-\beta)(1-SIT')^{-\beta/\alpha}}{\beta}}{t} \tag{12}$$

For an example of the application of these ideas, suppose that initially $\alpha=4.5$ and $\beta=.1$, which corresponds to a distribution where the mean number of mines is 4.5/(1-.1)=5. This distribution is the one labeled "prior" in Figure 5. If the individual mine threat is $t=.1$, then $SIT=g(4.5,.1,.1)=.392$. If it is desired to reduce SIT to .1 by sweeping, substitute $SIT'=.1$ in (12) to find $q=.211$. Sweeping must therefore be to the level .789. If $y=10$ mines are found and removed in the process of sweeping to that level, then (using (10)) $\alpha'=.211(4.5+.1(10))=1.1605$ and $\beta'=.211(.1)=.0211$. This is the "posterior" distribution of Figure 5. The average number of mines remaining is 1.186, and SIT is $g(1.1605,.0211,.1)=.1112$. The effect of the sweeping <u>and</u> the observation of the number of mines swept has been to reduce SIT, albeit not to exactly *.1*. A reduction is not inevitable; since SIT <u>increases</u> with the number of mines swept when $\beta>0$. In any case, the observation of the new SIT may prompt further sweeping, etc.

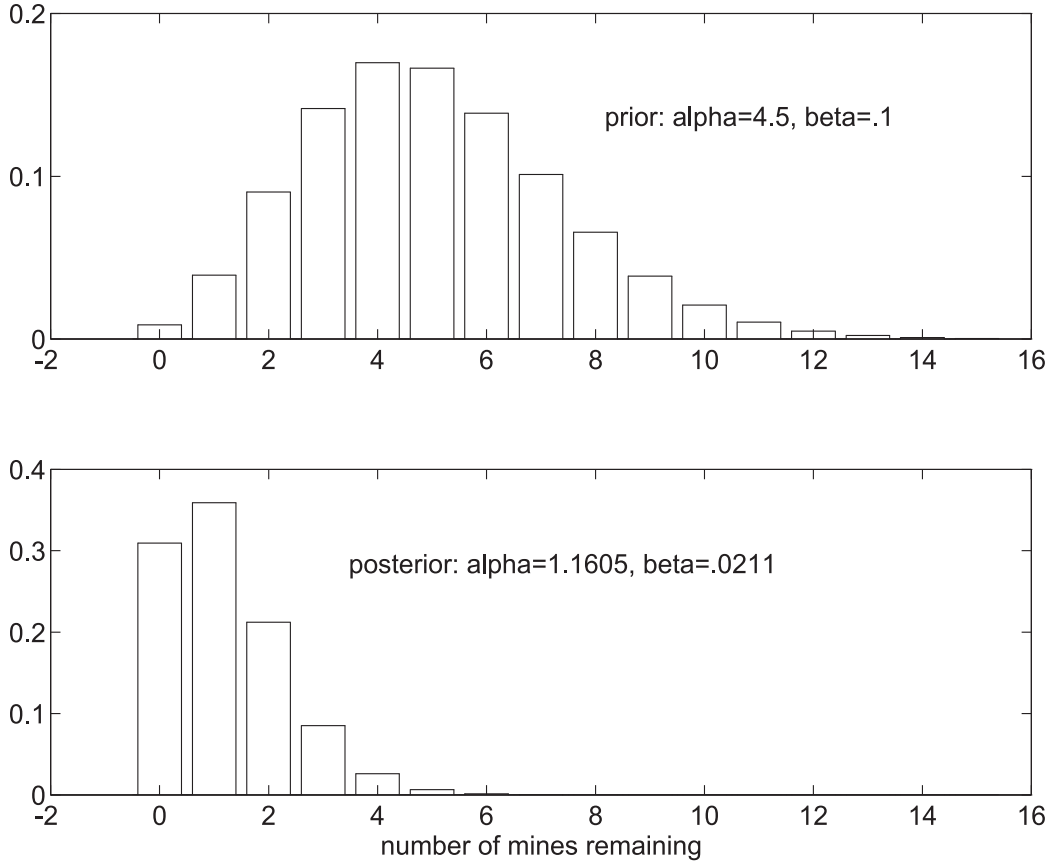

Figure 5: Distributions before and after minesweeping.

Only an extreme form of the above calculations has ever actually been used to aid the decision about whether there has been enough sweeping. The extreme form has α=1 and β=1-ε, where ε is extremely small. In that case M has the geometric distribution characteristic of the number of failures before the first success in repeated trials where the success probability is ε, a "diffuse prior" that places nearly equal (and therefore very small) weight on every nonnegative integer. In practice β is simply set to 1 in calculating α' and β'. The diffuse prior is a pessimistic assumption, since $E(M)$ is nearly infinite and SIT is nearly 1. The diffuse prior *apparently* has no need to make an initial assumption about the number of mines present, but only because α and β are built into the procedure instead of being a user input.

5.2 Optimal minesweeping in mixed minefields.

Minefields often consist of a mixture of mines of different types, partly because this forces the minesweeper to make repetitive passes with different sweep configurations. With several different configurations available, the question of how fixed resources should be divided among them arises. The answer, of course, depends on how much the sweeper knows about the mine populations. For the moment, assume that the number of mines of type i is independently Poisson with mean α_i; that is, suppose that Katz parameter β_i is zero in all cases. The minefield is to be swept and then transited. Assume that the first transitor will lethally actuate each remaining mine of type i with probability t_i, so that $z=\alpha_1 t_1+...+\alpha_m t_m$ is the average number of mines lethally actuated if there is no minesweeping. The aim of minesweeping is to reduce this number as much as possible, while not exceeding any of the resource constraints. Since the number of remaining mines is itself a Poisson random variable, the probability that the number of lethal mines is zero is SIT=1-exp(-z), but z itself will do for an objective because SIT is an increasing function of z.

Let x_j be the number of sweeps in configuration j, and let W_{ij} be the small probability with which each mine of type i is removed by each sweep of type j. The average number of times with which a mine of type i is removed is then $y_i \equiv \sum_{j=1}^{n} W_{ij} x_j$. Since there are many attempts at removal, each of which succeeds with a small probability, we take the probability that the mine survives all attempts to remove it as exp($-y_i$), the Poisson probability of no removals. Therefore the objective function is reduced by minesweeping to the level

$$z=\alpha_1 t_1 \exp(-y_1)+...+\alpha_m t_m \exp(-y_m) \tag{13}$$

which is to be minimized.

Let h_{jk} be the amount of resource k consumed by one sweep of type j, and let there be H_k units of resource k available over the minesweeping period. If there are K types of resource available, then the problem of selecting (x_1,...x_m) optimally reduces to the following optimization problem:

$$\text{minimize} \sum_{i=1}^{m} \alpha_i t_i \exp(-y_i)$$

$$\text{subject to } y_i = \sum_{j=1}^{n} W_{ij} x_j ; i = 1,...,m$$

$$\sum_{j=1}^{n} h_{jk} x_j \le H_k ; k = 1,...,K$$

$$\text{and } x_j \ge 0;\ j = 1,...,n.$$

This formulation depends for its simplicity on the random search assumption, but any convex function relating clearance level to the decision variables would do as well. Sheet "Optsweep" of workbook *MineWar.xls* uses Excel's Solver to solve the problem of allocating 4 resources to 7 sweep types to sweep 5 different types of mine. The reader may wish to experiment with it to see how the problem solution is sensitive to input data.

The case where Katz parameter $\beta_i \neq 0$ can also be formulated as an optimization problem, since the probability that the first transitor survives is still a product of factors raised to powers. Appendix B gives a GAMS formulation of such a problem that is otherwise the same as the one formulated on the "Optsweep" sheet (see exercise 10).

6. Countered Minefield Planning.

Section 4 deals with planning minefields when no countermeasures are expected. Section 5 deals with countering a minefield when no counter-countermeasures are expected. This section deals with planning counter-countermeasures when countermeasures are expected. The reader who fears that this sequence might go on indefinitely can take some comfort in noting that there is only one more section after this one.

There is a natural tendency for minefields to become less effective with time on account of sweeping and transits. Ship counters are one possible tactic for reducing this tendency. A mine on shipcount j will detonate when actuated only if j=1; otherwise, each actuation will decrease j by 1 until finally the mine is finally "ripe" (j=1). By mixing up the shipcounts of the mines, the minefield planner can achieve a minefield that is threatening for late transits, as well as for early ones. The problem of determining the ideal mixture of shipcounts is a good candidate for a computerized tactical decision aid. To make the ideas clear, we will describe the planning

problem in detail for an artificial minefield with only one type of mine and one type of transit, later making reference to software (ACMPM) that can handle multiple mine types without the artificial assumptions.

Suppose that transitors are all alike as far as actuating mines is concerned, but that one of N transitors (the "chief") is more important than the others. In fact, the sole object of the minefield planner is to threaten the chief regardless of where he appears in the sequence. If t_n is the threat (probability of damage) to the nth transitor, let t be the smallest of all these numbers. Since the chief may be anywhere in the transit sequence, the object is to make t as large as possible by cleverly setting the counts of the mines.

Suppose further that every transitor actuates each mine independently with probability A, and that the chief will be damaged with probability D, conditional on actuating a ripe mine. A mine initially set on shipcount j will be ripe just before the nth transit if and only if the first n-1 transits actuate it exactly j-1 times, a binomial probability. If P_{jn} is this probability, then

$$P_{jn} = \binom{n-1}{j-1} A^{j-1}(1-A)^{n-j}; 1 \le j \le n \le N \tag{14}$$

Note that P_{11}=1, since the number of combinations of 0 things taken 0 at a time is by definition 1. If the chief is transitor n, he will be damaged by this mine if and only if it is ripe, and if the chief both actuates it and is damaged by it. The probability of this is ADP_{jn}. If x_j is the number of mines set on shipcount j, and if all mines act independently, then the chief's probability of surviving all mines is

$$1 - t_n = \prod_{j=1}^{n} (1 - ADP_{jn})^{x_j}; 1 \le n \le N \tag{15}$$

The upper limit of the product in (15) is n because mines initially on shipcounts exceeding n cannot threaten the nth transitor. If there are a total of M mines available, then (15) is a product of at most M factors, each of is the probability of surviving one mine. A plot of t_n versus n is sometimes called a "threat profile".

We can now consider the problem of maximizing t, the minimum of all the numbers t_n, subject to the constraint that the variables x_j must not sum to more than M. Solutions can be surprising. One might think that there would have to be some mines on high initial shipcounts to guard against the possibility that the chief is late in the sequence. This is not generally true because A is generally considerably smaller than 1. If A is .2, for example, it would not be unusual to have a mine initially on shipcount 10 still be on shipcount 8 after 10 transits. If A is small enough, in fact, the best tactic may be to put all mines on shipcount 1. It is only in situations where actuation is likely, possibly because most transitors are actually minesweepers, that advanced shipcounts become attractive. Exercise 12 will give the reader a chance to experiment with this optimization problem.

The Analytical Countered Minefield Planning Model (ACMPM) is a program used to design countered minefields (Bronowitz and Fennemore, 1975) that avoids some of the artificial assumptions made above. ACMPM calculates P_{jn} for every possible cross-channel location of the mine before finally averaging to obtain the quantity used in (15), thus correctly reflecting the fact that mines don't change locations between transits. ACMPM also deals with a variety of mine types simultaneously, and includes resource constraints other than simple constraints on the number of mines. Still, ACMPM is similar in spirit and tendencies to the simple optimization

described above.

7. Two Person Zero Sum (TPZS) Games.

The measure/countermeasure cycle can be continued indefinitely, with each side thinking "If he thinks that I think that he thinks…" in trying to decide what to do next. TPZS games were invented to deal with that situation; they hold out the hope of basing actions on the enemy's known capabilities, rather than his presumed intentions. TPZS formulations have some problems of their own, as will be pointed out later, but they can sometimes be illuminating. The purpose of this section is not to be exhaustive, but merely to show some of the possibilities.

7.1 Matrix games.

Is it better to hunt for mines or to sweep them? Hunting has the advantage of working equally well regardless of the sensor type, mine count setting, or delay arming, since it is completely independent of the mine's sensors. On the other hand hunting usually has a comparatively small sweep width. For a simple analysis suppose that there are only two mine types (MAG and ACU) and three possible countermeasures (SMAG, SACU, and HUNT). The miner employs exactly one mine, having his choice of either type. The minesweeper elects to sweep or hunt without knowing the miner's choice, and the mine removal probability depends on the choices of both players. These six probabilities are shown in the matrix below. For the first two countermeasures the matrix entry is an actuation probability (we are assuming no mine counters or delay arms, so actuation is equivalent to removal). For the HUNT countermeasure the matrix entry is the probability of detecting the mine, which is again equivalent to removal. The sweeper is the maximizing player, so his three strategies are by convention shown as rows

	MAG	*ACU*
SMAG	*1.0*	*0.0*
SACU	*0.0*	*0.5*
HUNT	*0.3*	*0.3*

The solution of this game is that $x^*=(1/3,2/3,0)$ and $y^*=(1/3,2/3)$. x^* is the sweeper's optimal mixed strategy, with the three components being probabilities of choosing the three rows, and y^* is defined similarly for the miner. The value of the game is 1/3, the probability that the mine is removed. Note that MAG mines are used 1/3 of the time in spite of the fact that they are more easily swept than ACU mines, that the sweeper is more likely to do what he is bad at (SACU) than what he is good at (SMAG), and that the HUNT option is never used. If the HUNT detection probability were raised from .3 to .35, the sweeper would switch from exclusive sweeping to exclusive hunting and the value of the game would be .35. Some of these results are surprising, and it is hard to imagine discovering them by any means other than TPZS analysis. The practical problem with TPZS matrix analysis is usually in "scaling up" the solution to a problem with a practical number of alternatives. If the sweeper had three time periods available, for example, there would be $3^3=27$ strategies available even if each time period could have only a single type of activity. If each of 10 mines could be one of 3 types set on a count from 1 to 5, the miner would have $(3x5)^{10}$ strategies available, a very large number.

7.2 Concave-Convex games.

It is common for a decision problem to be easy to solve with only a few alternatives,

complicated for combinatorial reasons as the number of alternatives is increased, and then easy again in the limit as the set of alternatives becomes a continuum. The problem considered in section 7.1 is like this. The number of strategies available to the minesweeper increases fast if the number of sweep/hunt opportunities is increased, or if devoting part of a period to one activity and part to another is permitted, or if there are multiple units available for sweeping/hunting. This rapid increase destroys any hope of a matrix game solution, but analysis may still be possible. Suppose several identical units are present, let Y be the total number of unit-hours available during the time available for clearance, and suppose there are three sweeping/hunting activities. If y_j is the number of unit-hours devoted to activity j, then $y_1+y_2+y_3$ must not exceed Y for <u>any</u> feasible strategy. The continuum of alternatives is obtained by ignoring the scheduling details and concentrating entirely on this one constraint.

Assume for the sake of example that the numbers in the table of section 7.1 represent sweep rates as in section 5.2, rather than probabilities, with W_{ji} being the number in row j and column i. The average number of detections per mine of type i is $z_i \equiv W_{1i}y_1+W_{2i}y_2+W_{3i}y_3$, and, assuming again that search for mines is random, the probability of <u>not</u> removing a mine of type i is $\exp(-z_i)$. Let x_i be the probability that a mine is of type i and let $x \equiv (x_1,x_2)$. Then the objective function is $A(x,y) \equiv x_1\exp(-z_1)+x_2\exp(-z_2)$, the fraction of mines not removed, with the x-player maximizing and the y-player minimizing. Any probability distribution x is a strategy for the miner, any three nonnegative numbers adding up to Y is a strategy for the sweeper, and $A(x,y)$ determines the payoff. The advantage of this point of view is that $A(x,y)$ is concave in x and convex in y, so the game has a saddle point (Owen, 1982). Since the game has a saddle point, the sweeper has an optimal strategy, and this strategy y will assure that the miner has no cheap victories. The best y will therefore make the smallest of z_1 and z_2 as large as possible. Maximizing this minimum can be achieved with a small Linear Program (exercise 13); it turns out that Y should be split among the three tasks in the same proportions as y^* of section 5.1. Thus if two units were available for 24 hours each, Y would be 48 unit-hours and the game solution would be to spend 32 of them in ACU sweeps and the rest in MAG sweeps. No information about <u>how</u> the two units might be scheduled to achieve this is available; the game solution is a guide, rather than a detailed prescription.

Considerable generalization is possible. If the sweeper had several linear constraints involving y, rather than just one, the game would be still be solvable by Linear Programming. In fact it is interesting that the problem of minimizing $A(x,y)$ with x specified arbitrarily is a nonlinear program, whereas requiring that x be worst case (the game) is a simpler Linear Program. This is a rare instance where solving a game is simpler than solving the corresponding single-decision-maker optimization problem!

The assumption of random search assures that $A(x,y)$ is convex in y; any other assumption about minesweeping would do as well as long as it preserved the convexity of $A(x,y)$ in y. The game would be of a fundamentally more difficult type if $A(x,y)$ lost that property, since it might not have a saddle point.

The <u>number</u> of mines available need not be one. If M mines are available, then x_1+x_2 is simply required to be M and x_i is interpreted as "average number of mines of type i used by the miner". The number of types could also be increased without substantial computational penalty.

Generalization is more difficult in other directions. Suppose, for example, that type 3 mines are introduced as MAG mines on mine count 2. The number of activations of a MAG mine is a Poisson random variable with mean z_1 under the random search assumption, so the probability of activating a MAG mine 0 or 1 times is $\exp(-z_1)+z_1\exp(-z_1)$. This is not a convex function of z_1,

but there is an even more serious problem. Type 3 mines dominate MAG mines in the sense that the nonclearance probability is larger no matter what y is, so the miner would never use MAG mines on count 1 if count 2 were available. The best mine count is the largest number possible, and furthermore the best mine sensor would be none at all, since throwing the sensor away would surely prevent the mine from being swept! The problem is that the objective function "mine survival fraction" does not explicitly refer to the ultimate objective of mine warfare, which is to damage transitors. If optimizing mine counts or sensitivity settings is part of the miner's decision problem, then nonsensical results will be obtained unless the objective function is changed to reflect this ultimate objective.

7.3 Alternate objectives.

An obvious alternate objective would be *SIT*, the probability that the first transitor after sweeping is damaged. For example Washburn (1982) solved a game of this type where the optimal density of mines in the vicinity of a strait turned out to be nonuniform, becoming gradually smaller at distances far from the center of the constriction. But *SIT* confines applicability to situations where the number of transitors is small (one, to be precise). A minefield with a high *SIT* may still be ineffective in a practical sense, so one might wish to use some measure of the "staying power" of a minefield as the objective function.

It is easy enough to measure staying power: let the measure be the number of transitors damaged out of infinitely many, rather than out of just one. The threat to every transitor is important with this measure, rather than just the threat to the first. Unfortunately, using this measure is essentially the same as using "mine survival fraction", since damage is proportional to the number of mines not cleared if there is no danger of running out of targets. This attempt to "fix" *SIT* results in the problem described at the end of section 7.2.

The Navy's "Breakthrough" model (Sutter (1983)) fixes both the number of transitors and the amount of time available for clearance, and uses the objective function "fraction of mines that sink transitors". The miner's strategies include the mix of mine types and also a probability actuator setting β for each type. Making β=1 is not generally optimal because doing so makes the mines easy to sweep, and making β=0 is never optimal because even a mine that survives clearance would not damage anything. Selecting the right value for β is nontrivial, especially when the sweeper's options include both hunting, which is not affected by β, and sweeping, which is. The Breakthrough model is an analysis of this two-sided decision making situation. Breakthrough was originally intended as a mine clearance aid for use when dealing with a sophisticated miner, but it has actually enjoyed more use in force level studies where its ability to evaluate the effectiveness of a mix of minesweeping assets is useful.

What if the first transitor is damaged and the rest decide to detour around the minefield, go somewhere else, or take the damage as evidence that further minesweeping is necessary? If so is the miner to be given credit for delaying the arrival of the transitors? Probably he should be, since delay is often acknowledged to be one purpose of mining in the first place, but how much credit? Such questions can often be "finessed" in one-sided models. UMPM, for example, provides the user with a "stopped penetrator distribution" (section 4) without saying exactly what the user should make of it. This kind of finessing is not possible in a TPZS model, since the objective function must be explicit and either side may take advantage of any artificiality. Expect TPZS mine warfare analyses to continue to be occasional, rather than regular.

Exercises

1. Consider the ENWGS model of section 3. Suppose that M mines with action radius $W/2$ remain in a minefield of area A at the beginning of the fixed time interval δ that ENWGS uses to advance time, and consider an arbitrary ship that will travel some length L in the minefield over δ. As explained above, in the stochastic mode ENWGS compares a random number to WL/A to decide whether the ship is damaged by each of the M mines, subtracting one from M if any mine is struck. ENWGS also has a "deterministic" mode in which no random numbers are employed, the idea being to avoid the vagaries associated with randomness and assure reproducibility of results. The simplest deterministic model would be to replace all random numbers by .5, the midpoint of the interval [0,1]. Explain why this won't give satisfactory results, and suggest a better deterministic model. The principle should be that, since δ has no physical meaning and is chosen for reasons having nothing to do with the minefield, results should not depend strongly on what value happens to be chosen for δ.

2. The UMPM model of a minefield is equivalent to a Markov chain where the state is the number of ships remaining undamaged and where a transition corresponds to all of the remaining ships passing the next mine. Suppose that R_n^* is given by the ENWGS model with WL/A =.5, and that 2 ships attempt the transit of a minefield with 3 mines. Let x be a row vector storing the probabilities that 0, 1, or 2 ships remain undamaged, initially x=(0,0,1). What is the 3x3 transition matrix, what is x after three transitions, and what is the casualty distribution when two ships attempt to transit the minefield?

ans: x is (.656,.328,.016) after 3 transitions. The casualty distribution is just the reverse of x: (.016,.328,.656).

3. Consider revising UMPM so that there are ghost ships, as well as virgin ships, with ghost ships representing ships that have struck a mine, but which act like virgin ships as far as activating additional mines goes. When a ship activates a mine, the ship sinks with probability S, or else becomes/stays a ghost. Reconsider problem 2, but with S=.5. The state of the Markov chain will be (v,g), where v is the number of virgins (initially 2) and g is the number of ghosts (initially 0). You will need to consider transitions among six states: 20,10,11,00,01,02. Assume that R_{v+g}^* is the probability that the group of $v+g$ unsunk ships will actuate a mine, and that each of the unsunk ships is equally likely to be the actuator. What is the 6x6 transition matrix, what are the state probabilities after 3 transitions, and what is the casualty distribution? Ghosts count as casualties.

ans: If the state probability vector x is initially (1,0,0,0,0,0) with the states ordered as stated above, then it is (.0156, .2476, .1362, .2647, .2446, .0923) after 3 transitions. The casualty distribution is (.0156, .3838, .6006) for 0, 1, and 2 casualties. The transition matrix is

$$\begin{bmatrix} .2500 & .3750 & .3750 & 0 & 0 & 0 \\ 0 & .5000 & 0 & .2500 & .2500 & 0 \\ 0 & .1875 & .4375 & 0 & .1875 & .1875 \\ 0 & 0 & 0 & 1.000 & 0 & 0 \\ 0 & 0 & 0 & .500 & .500 & 0 \\ 0 & 0 & 0 & 0 & .3750 & .6250 \end{bmatrix}$$

4. In section 4.3, a claim is made about the size of a confidence interval for REP=10,000 replications. Verify it, and also give the size if REP=1000.

5. Consider the extreme example of section 4.2, but this time suppose that there are n=10 identical transitors and a single mine that is equally likely to be type 1 or type 2. What is the fully configured probability of a single casualty? What is UMPM's probability?

6. Using the methods of section 5.1, find an example where the results of minesweeping could increase *SIT* from its initial value.

7. Suppose that the prior distribution of M is as in section 5.1 with parameters α and β, and that sweeping is carried out in two stages. In stage i sweeping is carried out to level $1-q_i$ and y_i mines are removed from the minefield, i=1,2. One might determine the posterior distribution of the number of mines remaining by revising (α,β) twice, once for each sweep, or by arguing that the two sweeps together are equivalent to one sweep to the level $1-q_1q_2$ that removes y_1+y_2 mines. Show that both procedures give the same result.

8. Use sheet "Katz" of Excel workbook *MineWar.xls* to work this problem, if you have it available. Suppose that the prior distribution of mines is as in section 5.1 with parameters α=3 and β=.75, and that the individual mine threat is t=.1.

a) What is the initial *SIT*, and to what level must the minefield be swept to reduce *SIT* to the desired average level *SIT'*=.1?

b) If no mines are found, what is the post-sweep *SIT*, and what are the mean and standard deviation of the number of mines remaining? The mean should be .286.

9. Sheet "UMPM" of Excel workbook *MineWar.xls* implements the UMPM calculations against ten hypothetical transitors. Imagine that you are designing a minefield, and that you can control the "scale" input by changing the sensitivity of the sensor, as well as the "actuation prob" input, a number between 0 and 1 that represents the setting of a probability actuator. You are concerned about both the average number of casualties and the threat to the last (tenth) transitor. Can you find a choice of the two controllable parameters that makes both of these measures better? If so, what are the parameters and the resulting measures?

10. Appendix B formulates the problem of maximizing the survival probability S of the first transitor through a minefield with no counter-countermeasures after sweeping. The transitor must survive 5 different mine types, each of which has its own Katz parameters. It uses the same random search assumption for each mine as in section 4.2, except that the coverage ratio for type i mines is $y_i \equiv W_{i1}x_1+\ldots+W_{im}x_m$, where m is the number of sweep types (7 in the example). The decision variable x_j is the number of hours of sweeping of type j, and W_{ij} is a parameter measuring the efficiency of type j sweeping against type i mines.

a) Show that the objective function Z is -ln(S) provided that the initial numbers of mines of the 5 types are independent random variables and that $g(\alpha_i,\beta_i,q_it_i)$ determines SIT for each mine type.

b) Run the program and report the optimal value of S.

c) The objective function is a convex function of the decision variables. Explain why this is important.

11. The problem discussed in section 6 assumes that all transits have the same actuation probability A. Suppose that the chief actually has a different actuation probability B. How must equations (14) and/or (15) be revised?

12. Worksheet "ACMPM" of Excel workbook *MineWar.xls* implements equations (14) and (15), and invites you to determine the best minecount distribution for a given number of mines. See if you can find the distribution that maximizes the minimum threat when M=20, N=10, A=.4 and D=0.3. You may wish to take advantage of Excel's Solver feature, which is set up to find the best distribution if the requirement that the number of mines on each shipcount must be an integer is ignored. You may also wish to experiment with smaller but more realistic values of A

such as .1, in which case the benefits of advanced minecounts should be much smaller.

13. Confirm that the solution of the game discussed in section 7.2 is y=(32,16,0) by formulating and solving a Linear Program where the smallest z_i is maximized.

14. Suppose that the number of mines is a binomial random variable with parameters M and p. Formula (7) could be used to determine $x(k)$ for k=0,...,n. Argue that (7) is actually not required, since the inputs to UMPM can be modified instead. What is the modification?

15. Suppose that the number of mines M is a priori equally likely to be either 10 or 15, and that 7 mines are found in the process of sweeping to level .5. Use Bayes Theorem to find the probability that the original number of mines was 10, given the observed result.

16. Suppose that a single mine must be placed at x in the unit interval [0,1], while a transitor simultaneously selects a point y in the same interval in an attempt to pass safely by the mine. The transitor will be sunk if and only if $|x-y|\leq.2$. "Obviously" the location of the mine should be uniformly distributed over the interval and the game value is .4, since the mine can cover 40% of the interval. But not so fast…

a) Show that a clever choice of y would result in a sinking probability of only .2 against the uniform strategy.

b) Find a strategy for placing the mine that will always sink the transitor with probability 1/3 or more, regardless of y. Hint: The optimal distribution for x is discrete, not continuous.

c) Find a strategy for transiting (a distribution for y) that will result in being sunk with probability 1/3 or less, regardless of x.

The moral of this story is that end effects are potentially important.

17. There are two mines, each of which must be assigned a "count" of 0, 1, or 2 by the miner, a total of 9 possible joint assignments (these counts would normally be referred to as 1, 2, and 3, but it is convenient for the moment to begin counting from 0). The sweeper can sweep each mine either 0, 1, or 2 times, but the total number of sweeps cannot exceed 2 so there are only six joint possibilities. The sweeper wins if and only if both sides select different numbers for both mines, so the payoff matrix is a 9x6 matrix of 0's and 1's. This TPZS game models a situation where each mine is "ripe" only if its count is reduced to exactly 0 by sweeping, and where the sole transitor will encounter both mines. Solve the resulting 9 by 6 game.

18. Five ships transit a minefield of three mines. In terms of $R_1^*,\ldots,R_5^*$, as given by (5), what is the probability that exactly two of the ships will be damaged?

19. Find a formula analogous to (4) for the probability of a wasted fire: the probability that a mine at distance x from the channel centerline of n ships will actuate without doing any damage.

20. Sheet "Optsweep" of workbook *MineWar.xls* contains a formulation of the optimization problem considered in section 5.2.

a) Use solver to find the optimal distribution of minesweeping and the resulting SIT. The minimized *SIT* should be .2148, and the number of hours devoted to SHPMAG (ships making magnetic sweeps) should be 0.

b) The reason why SHPMAG is never used in the optimal sweeping plan is that each sweep requires 5 SHPHRS, and the SHPHRS resource is better spent on other sweep types. What would happen if that number were reduced from 5 to 3? You should find that the minimized *SIT* decreases to .2022, but what is the associated sweeping plan?

Solutions to Exercises

1. If δ is small, WL/A will be smaller than 0.5, and therefore no ship will ever be damaged, no matter how many small intervals there are. A better deterministic method might be to introduce a counter D for distance traveled. The counter would start at 0 and be incremented by L when the ship moves. When the counter exceeds A/W, the offending ship is damaged, and the counter reset to 0. Objections could also be made to the counter method; for example, no ship would ever be damaged if it traveled less than A/W in the minefield. There really is no simple deterministic equivalent to a Monte Carlo simulation.

2. The transition matrix is $P = \begin{bmatrix} 1 & 0 & 0 \\ .5 & .5 & 0 \\ 0 & .75 & .25 \end{bmatrix}$. The state vector x is (0.656, 0.328, 0.016) after 3 transitions. The casualty distribution is just the reverse of x: (0.016, 0.328, 0.656).

3. If the state vector x is initially (1,0,0,0,0,0) with the states ordered as stated in the problem, then it is (.0156, .2476, .1362, .2647, .2446, .0923) after 3 transitions. The casualty distribution is (.0156, .3838, .6006) for 0, 1, and 2 casualties. The transition matrix is

$$\begin{bmatrix} .2500 & .3750 & .3750 & 0 & 0 & 0 \\ 0 & .5000 & 0 & .2500 & .2500 & 0 \\ 0 & .1875 & .4375 & 0 & .1875 & .1875 \\ 0 & 0 & 0 & 1.000 & 0 & 0 \\ 0 & 0 & 0 & .500 & .500 & 0 \\ 0 & 0 & 0 & 0 & .3750 & .6250 \end{bmatrix}$$

4. The subject is "large sample confidence intervals for a population proportion". The halfwidth in general is $z_{\alpha/2}(p(1-p)/n)^{1/2}$, where n is the number of trials and p is the true sample proportion. Since $z_{\alpha/2}$ is 1.96 when α=.05, and since $p(1-p)$ cannot exceed 0.25, the halfwidth is approximately $(1/n)^{1/2}$. This is 0.01 when n = 10,000; i.e., to measure a proportion to within 0.01, one needs about 10,000 trials. When n = 1,000, the halfwidth is .03.

5. The fully configured probability is 0.5, since there will be one casualty if and only if the mine is type 1 (effective). The input to UMPM would have to be an actuation curve for which $A(x) = 0.5$ for all x, since the mine is equally likely to be effective or not. Therefore $R_n^* = 1-.5^n$, and UMPM's probability for n = 10 mines will be 0.999.

6. If $\beta > 0$, then SIT increases with the number of mines found, so any example with lots of mines found will suffice.

7. The first stage revised Katz parameters are $\alpha'=q_1(\alpha+\beta y_1)$ and $\beta'=q_1\beta$. The second stage parameters are $\alpha''=q_2(\alpha'+\beta' y_2)$ and $\beta''=q_2\beta'$. Substituting the first stage formulas into the second stage, we find that the two stages together are equivalent to a single stage where $q = q_1q_2$ and y_1+y_2 mines are removed.

8. The answers to part a are 0.650 and 0.911. The answers to part b are 0.028, 0.286, and 0.555.

9. There are many ways of accomplishing this. One is to change the scale from 30 to 40 and the actuation probability from 1/3 to .3, in which case the average number of casualties increases from 3.59 to 3.85, while the threat to the last transitor increases from 0.135 to 0.140.

10. Given the definitions of A_i and B_i, $(1\text{-}g(\alpha_i, \beta_i, q_i t_i))$ is the same thing as $(1+B_i)^{-A_i}$, each being the probability that the ship survives all mines of type i. Given the definition of Z and the assumption of independence between mine types, the probability of surviving all mine types is $S = \exp(-Z)$. The optimized value of Z is 0.844, which corresponds to a simple initial threat of 0.570. Convexity is important because this is a nonlinear optimization problem, and convexity guarantees a unique solution.

11. Substitute B for A in equation (15), but make no changes in equation (14).

12. The best integer solution found so far makes the threat at least 0.3263 regardless of where the chief appears, with the number of mines on each setting being (4, 4, 0, 10, 2, …). If you were able to intuit that solution, then congratulations to your intuition. If you find a better solution, please notify the author. If your solution isn't even that good, and if you found it using Solver, then you have learned something about Solver.

13. (solution stated in problem)

14. The modification is to simply multiply $d(x)$ by p and use M mines. In effect, p is a reliability for each mine.

15. Let M be the number of mines and Y be the number that are found. The binomial probability of getting 7 successes out of 10 trials with success probability .5 is P(Y=7|M=10) = .117. With 15 trials, the probability is P(Y=7|M=15) = 0.196. Since the events M=10 and M=15 are equally likely, according to Bayes theorem, P(M=10|Y=7) = 0.374.

16. In part a, the transitor's probability of being sunk is only 0.2 if it chooses either endpoint, as long as x is chosen uniformly. In part b, x should be equally likely to be either 0.15, 0.5, or 0.85. That way every point y is within range of at least one mine location. In part c, y should be equally likely to be either 0, 0.5, or 1. That way no mine single mine location can cover two transitor locations. The value of this continuous game is 1/3.

17. The payoff matrix of the 9x6 game is shown below, and the value is 2/3.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

18. The two damaged ships will be either the first and second, the first and third, or the second and third, so the answer is $R_5^* R_4^* (1-R_3^*) + R_5^* (1-R_4^*) R_4^* + (1-R_5^*) R_5^* R_4^*$. The same answer can be obtained with more effort by multiplying out the Markov chain probabilities.

19. The probability that the mine actuates, but does no damage, is $\{1-(1-A(x))^n\}\{1-D(x)/A(x)\}$. This is the product of the actuation probability and the probability of no damage, given actuation.

20. When the number of SHPHRS per SHPMAG sweep is reduced to three, the optimal number of SHPMAG sweeps increases form 0 to15.89, and the minimized SIT decreases to .2022.

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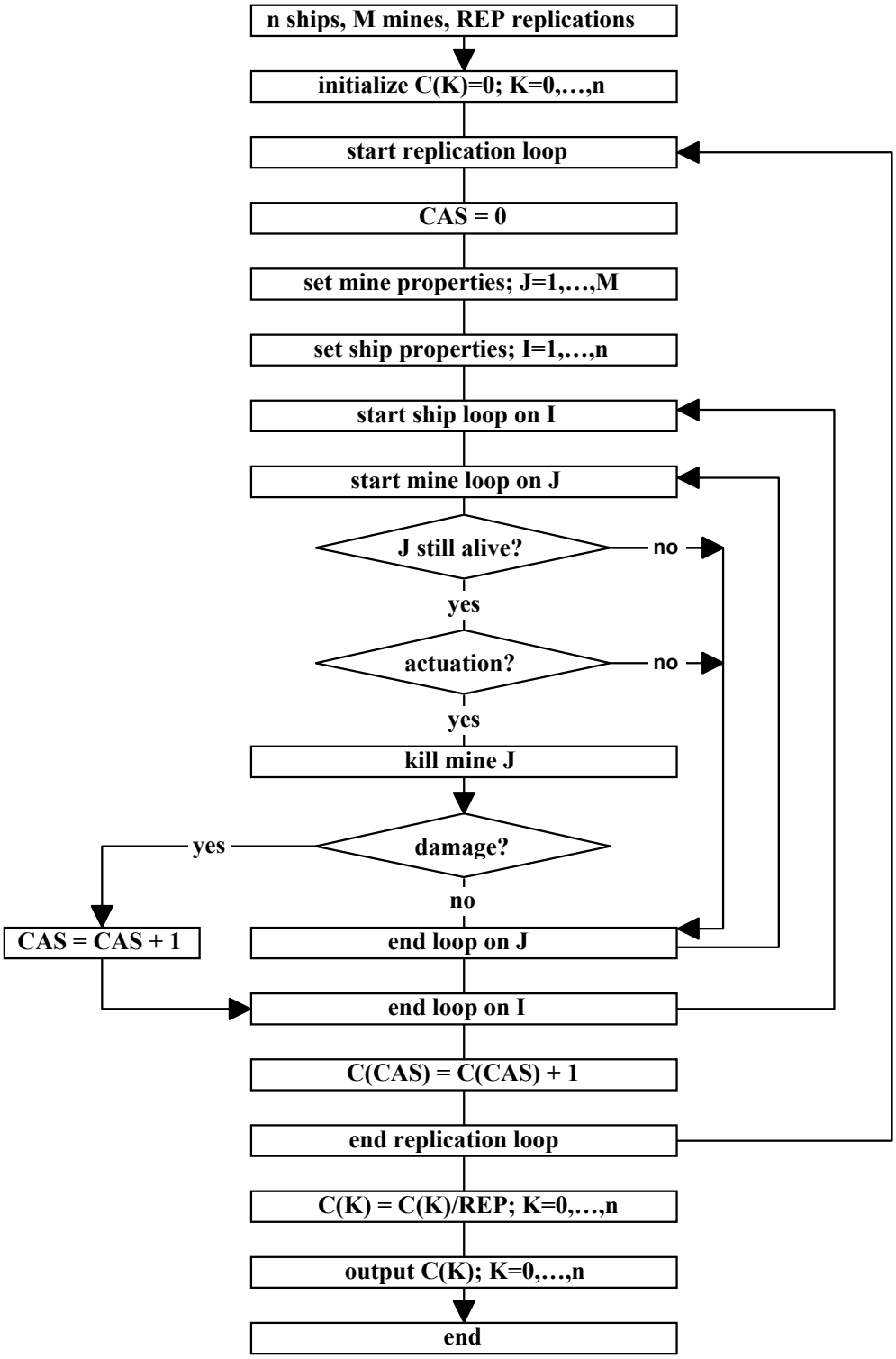
APPENDIX A: *Monte Carlo Simulation to Measure Casualty Distribution C( )*



APPENDIX B: GAMS Formulation of a Minesweeping Problem

```
$ TITLE NONLINEAR PROGRAM FOR ALLOCATING RESOURCES TO
MINESWEEPING
*** By Al Washburn, April 1994.  All values are arbitrary.
OPTION LIMROW=0,LIMCOL=0;
***BOT=bottom,MAG=magnetic,ACU=acoustic,PRS=pressure,TET=tethered***CNT=co
ntact,SLD=helicopter towed sled
***SHP=ship,EOD=EOD team,HEL=helicopter,CUT=cable cutting
***SHMGAC=ship using both magnetic and acoustic sweeps
SETS
  I mine type  /BOTMAG, BOTACU, BOTPRS, TETMAG, TETCNT/
  J sweep /SHPHNT, EODHNT, SHPMAG, HELMAG, SHMGAC, HELACU, HELCUT/
  K resource   /SHPHRS, HELHRS, EODHRS, SLDHRS/;
PARAMETERS
    ALPHA(I)  Katz parameter for each mine type
    /  BOTMAG 10
       BOTACU  8
       BOTPRS  6
       TETMAG  4
       TETCNT  2/
    BETA(I)   more Katz parameters
    /  BOTMAG  .1
       BOTACU  .3
       BOTPRS  .5
       TETMAG  .7
       TETCNT  .9/
    THREAT(I)  Unit threat per surviving mine
    /  BOTMAG  .02
       BOTACU  .01
       BOTPRS  .02
       TETMAG  .04
       TETCNT  .04/
    HRS(K)  clock hours available for resource k
     / SHPHRS  80
       HELHRS  48
       EODHRS  100
       SLDHRS  48/;
PARAMETER A(I) derived Katz parameter;
      A(I)=ALPHA(I)/BETA(I);
PARAMETER B(I) derived Katz parameter;
      B(I)=BETA(I)*THREAT(I)/(1-BETA(I));

TABLE W(I,J)  sweep rate of column J versus row I
              SHPHNT  EODHNT  SHPMAG  HELMAG  SHMGAC  HELACU  HELCUT
  BOTMAG       .02     .01     .05     .10     .01     .00     .00
  BOTACU       .02     .01     .00     .00     .01     .05     .00
```

```
  BOTPRS     .02      .01      .00      .00      .00      .00      .00
  TETMAG     .03      .00      .05      .10      .01      .00      .08
  TETCNT     .04      .00      .00      .00      .00      .00      .10;


TABLE H(K,J) clock hours of resource k per hour on task of sweep j

          SHPHNT  EODHNT  SHPMAG  HELMAG  SHMGAC  HELACU  HELCUT
  SHPHRS   4.0      1.0      5.0      0.0     16.0      0.0      0.0
  HELHRS   0.0      0.0      0.0     12.0      0.0     12.0     14.0
  EODHRS   1.0      4.0      0.0      0.0      0.0      0.0      1.0
  SLDHRS   0.0      0.0      0.0      2.0      0.0      2.0      0.0;
VARIABLES
      X(J)  total hours on task for sweep type j
      Q(I)  probability that a mine of type i will not be swept
      Z     negative of the log of the initial transit survival probability;
POSITIVE VARIABLE X;
EQUATIONS
      OBJ     define objective function
      MINESURV(I)  define q(i)
      RESOURCE(K)  enforce resource restrictions on clock hours;
OBJ..         Z =E= SUM(I,A(I)*LOG(1+B(I)*Q(I)));
MINESURV(I)..  Q(I)-EXP(-SUM(J,W(I,J)*X(J))) =E= 0;
RESOURCE(K)..  SUM(J,H(K,J)*X(J)) =L= HRS(K);
MODEL MINESWEEP /ALL/;
OPTION NLP=MINOS5;
SOLVE MINESWEEP USING NLP MINIMIZING Z;
DISPLAY X.L;
PARAMETER SIT simple initial threat;
SIT = 1-exp(-Z.L);
DISPLAY SIT;
```